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A MULTI-HIERARCHY FACILITY LOCATION PROBLEM WITH ROUTING UNDER DEMAND UNCERTAINTY: AN APPLICATION TO RELIEF DISTRIBUTIONS

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Demand fluctuation, known as demand uncertainty, usually occurs under several circumstances not only in ordinary logistics firm but also humanitarian logistics. An objective of this study is to tackle the facility locations and allocations under demand uncertainty in a case study for a relief distribution. The demand uncertainty is handled by robust counterpart in Robust Optimization (RO) called an ellipsoidal uncertainty set, which is a novel approach that has never been fully applied so far solving on a multi-facility location network. Herein, we analyze and compare three distinct network designs to find the most cost efficient and robust network. The best model is selected through continuous improvement by routing optimization, leading to delivery cost reduction.

Key Words: *multi-hierarchy, demand uncertainty, facility locations, Robust Optimization*

1. INTRODUCTION

Humanitarian logistics have been indicated as significant issues in several terms of natural disaster operations and management. Recently, researchers have extensively worked on the most appropriate location of medical centers and shelters to provide the evacuees quick access. Lin *et al.*2) found that prioritizing items for delivery and an extensive time period are important factors in humanitarian logistics. Accordingly, to improve logistics efficiency, they proposed the location of temporary depots to be around the disaster-affected area, along with the required vehicles and resources. In contrast, our study intends to design the depot locations by considering the cost efficiency and whether demand is met. Real situations usually involve uncertainty, that is, parameter fluctuation. Holguín-Veras3) has inferred from the Tohoku experience that, in order to improve future response efforts, disaster planners should design worst case scenarios from small disasters to massive destruction. Therefore, this study also stresses the importance of such improvements in case the demand becomes uncertain. Furthermore, we aim to improve the network by considering the practical level of vehicle routing problems after obtaining the locations at a strategic level are obtained under demand uncertainty, setting up a two-stage facility location network with a route planning problem. Baron and Milner4) have carried on the similar works on facility location by using the Robust Optimization approach. The potential of box uncertainty set and ellipsoidal uncertainty set was investigated to solve facility location problems by varying the size both box and ellipsoid. They found that the box uncertainty model performs poorly with respect to balancing robustness with the model objective function. However, their model only considered a single layer of facility location. On the contrary, this study focuses on the fully multi-hierarchy facility location network, which is more complicated but more realistic. Min *et al.*5) found that there is a difference between location with routing and classical location: while the first one serves customers through tours, the second one serves the customers through radial trips.

In such a background, this study attempts to contribute to the previous research by considering the worst case scenarios for multi-facility location problems under demand uncertainty. Accordingly, the main objective of the study is to tackle the facility location and allocation problem with demand uncertainty by using an ellipsoidal uncertainty set approach, which has never been fully applied so far. This allows to solve the multi-hierarchy facility location network problem, and subsequently building vehicle routing from the allocations derived from location problem with demand uncertainty. The objectives of the study can be summarized as follows:

1. To apply ellipsoidal uncertainty set, a novel approach to solve fully multi-hierarchy facility location network problems.
2. To manage the facility location problem in the context of both deterministic and uncertain demand, and subsequently compare solutions' robustness of five

demand scenario assumptions.

3. To evaluate the total delivery cost efficiency on different networks. Accordingly, we analyze the model on three networks differing in structures and truck sizes.
4. To select the best network and improve it by making practical uses of routing.

The expected results from the facility location model with routing are defined as follows:

1. To search the appropriate locations of depots for relief items' distribution in Miyagi prefectures.
2. To allocate the transportation link flow at each network configuration.
3. To minimize the total delivery cost which includes transportation cost, facility opening cost, and transshipment cost.
4. To seek a practical routing to reduce the total delivery cost.

In upcoming sections, we state the problem, and present model structure and assumptions, as well as results and conclusions.

2. PROBLEM STATEMENT

The 2011 Tohoku' Earthquake in Japan, represents our case study. We focus on Miyagi prefecture, the most affected area with a huge number of evacuees. As it is widely known, the 2011 Tohoku earthquake and tsunami had enormous impacts both as a humanitarian crisis, while also leaving economic aftermath. Moreover, the situation generated the several costs during and after disaster like reconstruction cost, rescue cost, logistics cost, etc. The logistics cost is quantified by Nagurney *et al.*[6] as approximately 80 percent of overall operation responding costs. Therefore, the cost efficiency should be one of the many aspects to be considered. Because of this, this study would like to examine the logistics cost efficiency. An improved supply distribution cost can reduce the expenditure for the whole operation cost during the amelioration period. A bottle of water is considered to be a requisite item for preliminary succor. Although total delivery cost minimization is not the only aspect to be considered in humanitarian logistics, it is a good criterion to compare the results for distinct network systems.

In fact, post disaster circumstances usually involve fluctuations in the number of evacuees and imprecise predictions. This study aims to handle the facility location problems or mixed integer problems under demand uncertainty. The methodology to deal with this demand fluctuation is Robust Optimization. Snyder[7] surveyed the facility location under uncertainty. The author classified several papers by their approach to uncertainty. Those are stochastic location problems and robust location problems. However, there is no multi-facility location problem solved through robust counterpart. Accordingly, we attempt to address the problem by employing a robust counterpart of the ellipsoidal uncertainty set, which is a novel approach that has never been fully applied so far to solve on multi-facility location problems.

In addition, according to the definition of solitary facility location problem, the classical location serves customers through radial trips. They do not serve reasonably through tours as in more realistic operational circumstances. Therefore, the route building is conducted to improve the distribution network onward.

3. MODEL STRUCTURE AND DATA ASSUMPTIONS

3.1. Model structure

The problem is designed for three different network structures. We categorized the three networks based on network configurations and dispatched truck sizes. The first network element consists of the locations with serviceable supports (suppliers). The second network element is the central relief depot in case of double hierarchies. The third element is constituted by the relief depots for double hierarchy and the relief depot in case of single hierarchy. The latter two network locations are unknown and need to be defined in the most efficient way. Finally, areas subject to a natural disaster are called shelters, which can be defined known locations as demands. The transportation truck sizes are 10-ton trucks and 4-ton trucks. Networks specification is described below.

（1）Network 1

This network is determined by three network elements: suppliers, relief depots and shelter demands. The relief depot candidate sites are located inside the affected areas. The relief items are dispatched from suppliers to relief depots by using

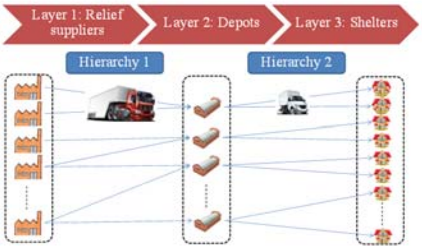


Fig.1 The single-hierarchy network framework and 10-4-ton truck delivery

10-ton trucks. Then, 4-ton trucks are used for portage of the relief items from relief depots to shelter demands.

（2）Network 2

This network is determined by four network elements: suppliers, central relief depots, relief depots and shelter demands. The central relief depot candidates are supposed to be located inside the affected areas. The 10-ton trucks are meant to transport relief items from suppliers to central relief depots and from central relief depots to relief depots. Then, relief items are carried from relief depots to shelter demands by using 4-ton trucks.

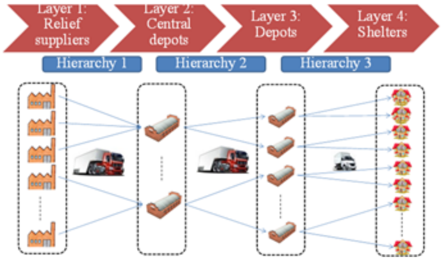


Fig.2 The two-hierarchy network framework and 10-10-4-ton truck delivery

（3）Network 3

This network is a duplicate structure of network 2 in terms of the number of network configurations and their locations. However, there is a difference in terms of truck size, as here the transportation from central relief depots to relief depots is assigned to 4-ton trucks instead of 10-ton trucks.

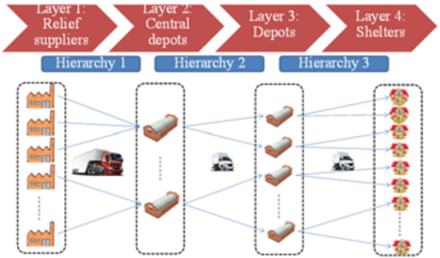


Fig.3 The two hierarchies network framework and 10-4-4-ton truck delivery

3.2. Shelter locations and number of evacuees

This study used data published in the Miyagi Prefectural Government Report[8)]. The number of evacuees (Figure 4) is considered to estimate the amount of relief items demanded and their locations. We classified locations into zones distinguished by different colors in Figure 4. The data indicated that the number of evacuees was very high in flat areas and the capital city. The highest demand, approximately 70,000, was recorded in Sendai. Therefore, we divided Sendai into five demand sites. Although the number of victims in Ishinomaki was also high, the demand site was not subdivided because the flat area is quite small. Hence there is no significant impact in terms of distance. Further, there were relatively less evacuees on the top and bottom sites located near Sendai city. The lowest number of evacuees (5 evacuees) was recorded in the Marumori town where connects under Sendai. Exempt these mentions before a number of evacuees are quite small when comparing with Sendai and Ishinomaki. These amounts are some hundreds of people for each shelter.

The study not only examines a single demand model but also multi-demand scenarios are examined. These demand scenarios are used in order to analyze the sensitivity of three different network structures. The demand is determined for five scenarios which deviate from the historical case in both sides optimistic and pessimistic. These five demand scenarios are separated as less than historical demand by 20 percent (S1) and 10 percent (S2), historical demand (S3), and more than actual demand by 10 percent (S3) and 20 percent (S4) respectively.

3.3. Supplier locations and facility locations

This section specifies the network element locations. These locations are identified by latitude and longitude coordinates. Our study assumes that locations are easily accessible by expressway or main road. Absent significant distance, locations might also be a group of sites. First, the locations of suppliers are determined. The Miyagi Prefectural Government Report[8)] shows that in Japan there are 29 sites available for serving the affected areas with goods after the disaster. However, we grouped these 29 sites into nine sites of suppliers by using the adjacent zone ideal. The locations of supplier locations are illustrated in Figure 5(a).

Then, the locations of candidate central depots and candidate depot locations are defined. The assumption is that the central depots are inside the affected areas or in Miyagi prefecture. These are assigned to cover the areas of the next layer or depots. The central depots should respond to the next layer necessities by considering the zoning characteristics. The locations of candidate central depots are illustrated in Figure 5(b). Then, the candidate depot locations are indicated in 11 places. These locations are specified by clustering zoning areas and demand size in Figure 5(c). As mentioned in the previous section, the different colors of depot candidates are supposed to correspond to different demand zones.

3.4. The total delivery cost

We assume that total delivery costs can be separated into three parts: travel cost, facility opening cost and transshipment cost. Firstly, the travel cost is measured by the fuel consumption rate based on 4-ton trucks and 10-ton trucks. Moreover, the travel cost also includes driver's salary under the working time limitation, purchase truck cost per day and vehicle insurance per day. Secondly, the facility opening cost is determined.

This cost is measured as the average price of building rental for firm, and it differs depending on the zone. Finally, the transshipment cost is set about three times the value of an ordinary business firm (15,000 yen per ton per day) for loading and unloading goods.

3.5. The travel cost

This cost changes according to the distance and energy consumption rate. At the beginning, the energy consumption rate for 4-ton trucks and 10-ton trucks was set to 7.69 and 11.54 yen per kilometer respectively. The travel cost is computed below and considers travel cost from length, driver salary, purchase truck cost and insurance cost. The driver cost is defined per hour of driving with an upper limit of eight hours

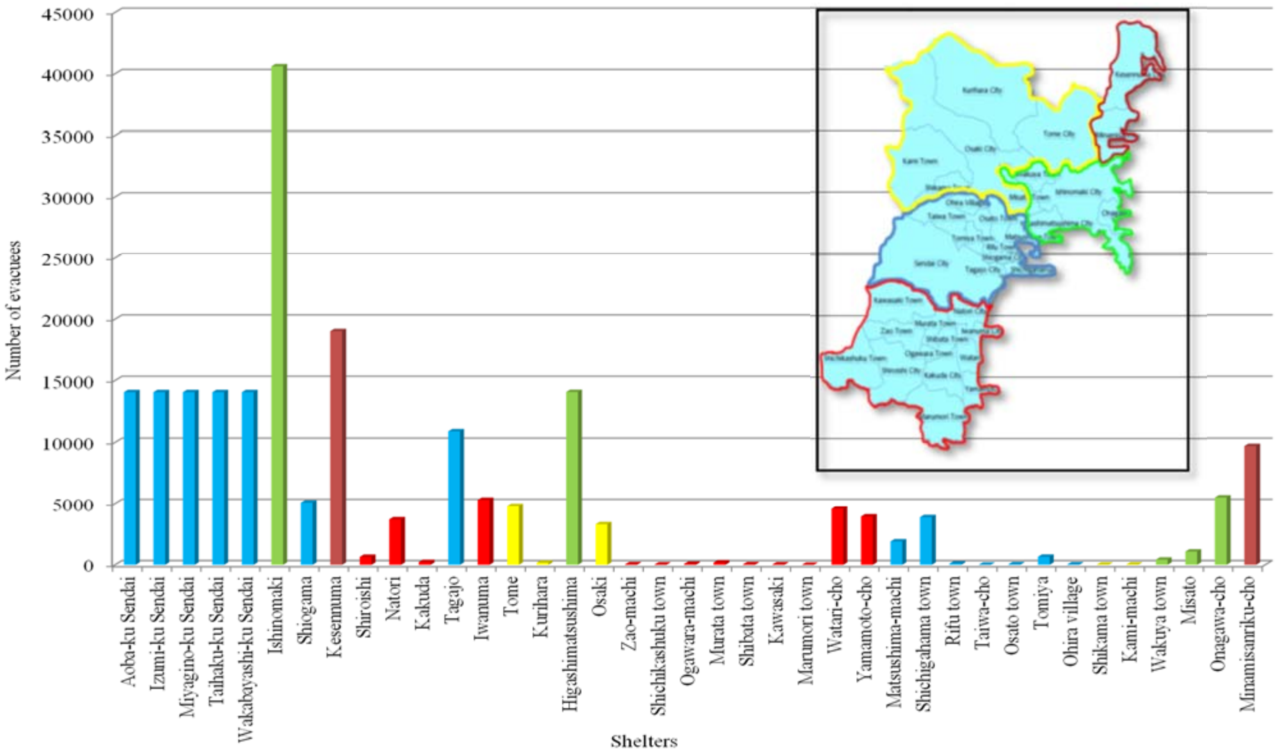
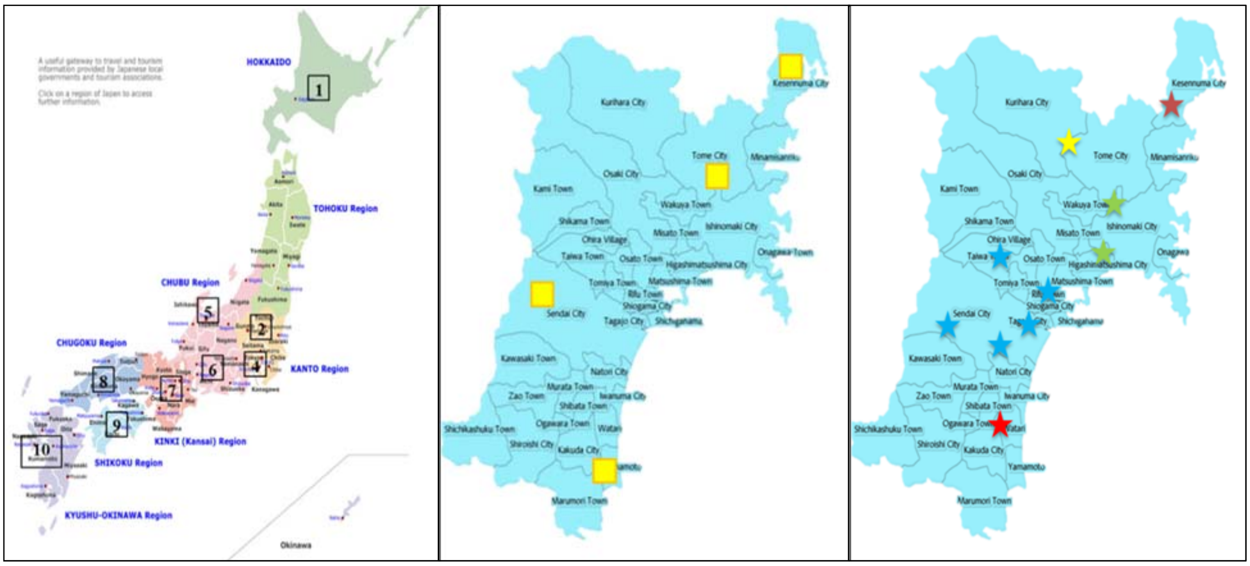


Fig.4 The amount of demand in each shelter and zone clustering (in the map)



(a) Supplier locations (b) Central depot locations (c) Depot locations

Fig.5 The supplier and facility locations per day. Purchase truck cost and vehicle insurance cost are divided into one day units.

a. Large truck size (10-ton)

*driver salary = 1,250 yen per hour
*purchase truck cost = 2,063 yen per day
*vehicle insurance cost = 1,032 yen per day

b. Small truck size (4-ton)

*driver salary = 1,000 yen per hour
*purchase truck cost = 1,375 yen per day
*vehicle insurance cost = 688 yen per day

3.6. Facility opening cost

The facility opening cost or fixed cost includes construction cost, electricity cost and water supply cost as well as the rental building cost based on websites in @Nifty[10]. This cost is measured as the average price of a storage building rental in each area of central depot candidates. This implies that the average price of building rental is a function of the storage size. However, several severe destructions including road disruptions in the affected areas after the disaster need to be taken into account. Therefore, it becomes very difficult to determine the suitable locations for candidate central depots and depots. Moreover, after the disaster there is also a lack of necessary resources like human resources, electricity supplies, water supplies, fuel energy and particularly land areas. Therefore, the opening cost for the facility location is set 10% higher than the original.

3.7. Transshipment cost

Transshipment cost is assumed to be approximately 15,000 yen per ton per day. This cost is three times the typical business firm cost for loading or unloading of goods at facility sites. This cost will generate when the set of facility sites is selected to operate. This cost includes salary, bonus, social insurance and pension saving of human resources. This value is obtained from the firm Logistic Behavior Survey (PWRI)[9].

4. MATHEMATICAL FORMULATION

Indices

I: Set of the supplier nodes (i) (i=1,2,3…I)
J: Set of the candidate central depots (j) (j=1,2,3…J)
K: Set of the candidate depots (k) (k=1,2,3…K)
L: Set of the demand nodes or shelters (l) (l=1,2,3…L)
TS: Set of the truck size (s)

Notations

$x_{ij}^1, x_{jk}^2, x_{kl}^3$: The flow of items from i and j, j and k, k and l
C_j^1 : The capacity at the candidate central depots j (j=1,2,3…J)
C_k^2 : The capacity at the candidate depots k (k=1,2,3…K)

Sets of parameters

S_i : The amount of items at the supply i
D_l : The demand at the affected area l
$c_{ij}^1, c_{jk}^2, c_{kl}^3$: The travel cost between i and j, j and k, k and l
f_j^1, f_k^2 : The opening depot cost at j and k
tc_j^1, tc_k^2 : The transshipment cost at j and k
$v_{ij}^1, v_{jk}^2, v_{kl}^3$: The capacity of truck between i and j, j and k, k and l
$w_{ij}^1, w_{jk}^2, w_{kl}^3$: The maximum working time of drivers between i and j, j and k, k and l
$d_{ij}^1, d_{jk}^2, d_{kl}^3$: The distance between i and j, j and k, k and l
$t_{ij}^1, t_{jk}^2, t_{kl}^3$: The travel time between i and j, j and k, k and l
E_s : The energy consumption rate of truck sizes
DS_s : The driver salary of truck size s
T_s : The truck cost of truck size s

4.1. Objective function

Classical location problem

$$min\left\{\sum_{j=1}^{J}\sum_{i=1}^{I} c_{ij}^1 x_{ij}^1 Y_j + \sum_{k=1}^{K}\sum_{j=1}^{J} c_{jk}^2 x_{jk}^2 Y_j Z_k + \sum_{l=1}^{L}\sum_{k=1}^{K} c_{kl}^3 x_{kl}^3 Z_k + \sum_{j=1}^{J} f_j^1 Y_j + \sum_{k=1}^{K} f_k^2 Z_k + \sum_{j=1}^{J} tc_j^1 Y_j + \sum_{k=1}^{K} tc_k^2 Z_k\right\} \tag{1}$$

When

$$c_{ij}^1 = \left(E_s d_{ij}^1\right) + \left(DS_s t_{ij}^1\right) + T_s \tag{2}$$

$$c_{jk}^2 = \left(E_s d_{jk}^2\right) + \left(DS_s t_{jk}^2\right) + T_s \tag{3}$$

$$c_{kl}^3 = (E_s d_{kl}^3) + (DS_s t_{kl}^3) + T_s \tag{4}$$

Decision variables

$$Y_j = \begin{cases} 1, \ if\ central\ depots\ is\ located\ at\ j \\ 0, \ otherwise \end{cases} for\ j\ \varepsilon\ J \tag{5}$$

$$Z_k = \begin{cases} 1, \ if\ depots\ is\ located\ at\ k \\ 0, \ otherwise \end{cases} \quad for\ k\ \varepsilon\ K \tag{6}$$

Subject to

$$\sum_{j=1}^{J} x_{ij}^{1} \leq S_i \tag{7}$$

$$\sum_{i=1}^{I} x_{ij}^{1} \geq \sum_{k=1}^{K} x_{jk}^{2}\, Y_j \tag{8}$$

$$\sum_{k=1}^{K} x_{jk}^{2} \leq C_j^{1} Y_j \tag{9}$$

$$\sum_{j=1}^{J} x_{jk}^{2} \leq \sum_{l=1}^{L} x_{kl}^{3}\, Z_k \tag{10}$$

$$\sum_{l=1}^{L} x_{kl}^{3} \leq C_k^{2} Z_k \tag{11}$$

$$\sum_{k=1}^{K} x_{kl}^{3} \geq D_l \tag{12}$$

$$\sum_{j=1}^{J} x_{ij}^{1} \leq v_{ij}^{1} \tag{13}$$

$$\sum_{k=1}^{K} x_{jk}^{2} \leq v_{jk}^{2} \tag{14}$$

$$\sum_{l=1}^{L} x_{kl}^{3} \leq v_{kl}^{3} \tag{15}$$

$$\sum_{j=1}^{J} t_{ij}^{1} \leq w_{ij}^{1} \tag{16}$$

$$\sum_{k=1}^{K} t_{jk}^{2} \leq w_{jk}^{2} \tag{17}$$

$$\sum_{l=1}^{L} t_{kl}^{3} \leq w_{kl}^{3} \tag{18}$$

$$x_{ij}^{1}, x_{jk}^{2}, x_{kl}^{3} \geq 0 \tag{19}$$

$$\mathrm{Y_j},\ \mathrm{Z_k} \in \{0,1\} \text{ for all j and k} \tag{20}$$

The objective function (1) minimizes the total delivery cost as the sum of transportation cost, facility cost and transshipment cost. Formulation (2), (3) and (4) represent travel cost parameter functions including travel cost and operation cost. Y_j and Z_k in (5) and (6) are decision variables to locate the central depots j and depots k. Note that (1) is completely performed for network 2 and network 3 through both decision variables Y_j and Z_k in (5) and (6), respectively. However, there are no central depots j in network 1, therefore the links from suppliers i to, respectively, depots k and to demand l, are the same. Constraint (7) guarantees that total flows from suppliers i to central depots j are not higher than the amount of serving goods at suppliers i. Constraint (8) ensures that the sum of link flows from i to j does not exceed the total availability of goods at opening central depots j. Constraint (9) limits the total amount of link flows from j to k to the capacity of opening central depots j. Constraint (10) limits the sum of link flows from j to k to the availability of goods at depots k. Constraint (11) ensures that the total amount of link flows from depots k to demand l must not be higher than the capacity of next network configuration or depots k. Constraint (12) confirms that the total amount of goods from depots k satisfies demand l. Constraints (13), (14), and (15) prohibit that the amount of a commodity exceeds the maximum truck volume. Constraints (16), (17), and (18) limit the total amount of driving hours to the maximum working time. Constraint (19) confirms that each link flow from site i to j, j to k, and k to l needs to define some amount of goods. Finally, constraint (20) specifies that both decision variables Y_j and Z_k are binary variables, taking value 1 if the facility is located at sites j and k, and 0 otherwise.

Decision variables

$$R_{kl} = \begin{cases} 1,\ if\ shelter\ \ l\ is\ delivered \\ from\ \ depots\ \ k\ \ by\ truck\ \ s \qquad for\ \ k\ \varepsilon\ K, l\ \varepsilon\ L \\ \qquad 0,\ otherwise \end{cases} \tag{21}$$

Location problem with routing

$$min\left\{\sum_{j=1}^{J}\sum_{i=1}^{I} c_{ij}^{1} x_{ij}^{1}\, Y_j + \sum_{k=1}^{K}\sum_{j=1}^{J} c_{jk}^{2} x_{jk}^{2}\, Y_j Z_k + \sum_{l=1}^{L}\sum_{k=1}^{K} c_{kl}^{3} x_{kl}^{3}\, Z_k R_{kl} + \sum_{j=1}^{J} f_j^{1} Y_j + \sum_{k=1}^{K} f_k^{2} Z_k + \sum_{j=1}^{J} tc_j^{1} Y_j + \sum_{k=1}^{K} tc_k^{2} Z_k \right\} \tag{22}$$

Additionally subject to

$$\sum_{l=1}^{L} R_{kl} \geq 1 \tag{23}$$

$$\sum_{k=1}^{K} x_{kl}^{2} Z_k \sum_{l=1}^{L} R_{kl} \leq v_{kl}^{3} \tag{24}$$

$$\sum_{l=1}^{K} R_{kl} - \sum_{l=1}^{L} R_{kl} = 0 \tag{25}$$

This part refers to the second stage of this study that is improvement through routing. The variables in the location problem, namely locations and allocations, were determined in the primary stage. In other words, the decision Y_j, Z_k and variables $x_{ij}^1, x_{jk}^2, x_{kl}^3$ in (22) are derived from the optimization in (1) under demand uncertainty (26, 27, and 28). Then, the decision Y_j, Z_k and variables $x_{ij}^1, x_{jk}^2, x_{kl}^3$ in (27) turn into parameters to plug in (22). Subsequently, the variable R_{kl} in (21) is added to generate the tours at the third hierarchy. The objective function (22) minimizes the total delivery cost of location with routing problem. Constraint (23) ensures that the delivery route is linked to the opening depots. Constraint (24) fixes the capacity of trucks at depots. Constraint (25) requires that the trucks ought to leave every point that they visited.

4.2. Mathematical with Robust Formulation by using Robust Counterpart

This part of the study focuses on the multi-source and multi-layer aspects of the facility location problem with uncertainty demand by considering the ellipsoidal uncertainty set within the robust optimization approach. *Ben-Tal and Nemirovski*[15] considered the ellipsoidal uncertainty set with linear programming. *Kouvelis and Yu*[16] discussed the robust discrete optimization and its applications. They proposed an approach to find a solution that minimizes the worst case performance under a set of scenarios for the data. *Bertsimas and Brown*[17] proposed a methodology to construct uncertainty sets for robust linear optimization based on decision maker risk preferences. *Josef*[18] gave an overview of the state-of-the-art and recent advances in mixed integer optimization to solve planning and design problems in industry processes. Stochastic programming for continuous LP problems is now included in most optimization packages, and there is encouraging progress in the fields of stochastic MILP and robust MILP. *Ben-Tal, Bertsimas and Brown*[19] proposed a soft robust model for optimization under ambiguity.

$$\text{Ellipsoidal: } U = \left(\frac{(D-\bar{D})^t}{\delta\times(D-\bar{D})}\right)^2 \le \rho^2 \tag{26}$$

$$\begin{aligned} min \Bigg\{ & \sum_{j=1}^{J}\sum_{i=1}^{I} c_{ij}^1 \bar{x}_{ij}^1 \bar{Y}_j + \sum_{k=1}^{K}\sum_{j=1}^{J} c_{jk}^2 \bar{x}_{jk}^2 \bar{Y}_j \bar{Z}_k \\ & + \sum_{l=1}^{L}\sum_{k=1}^{K} c_{kl}^3 \bar{x}_{kl}^3 \bar{Z}_k + \sum_{j=1}^{J} f_j^1 \bar{Y}_j + \sum_{k=1}^{K} f_k^2 \bar{Z}_k \\ & + \sum_{j=1}^{J} tc_j^1 \bar{Y}_j + \sum_{k=1}^{K} tc_k^2 \bar{Z}_k \Bigg\} \end{aligned} \tag{27}$$

$$\sum_{k=1}^{K} \bar{x}_{kl}^3 \ge \overline{D_l} \tag{28}$$

This study focuses on the demand uncertainty parameter, which deviates from the historical value of the uncertain parameters. Demand uncertainty is expanded in the region of the ellipsoidal uncertainty set. The demand parameter in (26) is D while $(\bar{D})$ is the demand deviating from historical values. Given the uncertain demand $\bar{D} \in R^d$, we consider the sets around the historical values $D \in R^d$, where δ is a scaling parameter for the ellipsoid radius and ρ^2 is the constraint. The size of the ellipsoid can increase by the ellipsoid radius δ or the constraint ρ^2. Increasing the size of the ellipsoid will make the model more robust against (more) uncertainty, however at the cost of a worse solution. Moreover, if the size of the ellipsoid becomes too large then the model might become infeasible. In accordance with the solution feasibility, we set δ equal to 1/150 – 1 and use ρ^2 equal to 1 to restrict the region around historical demand. We determine that the interval range of demand $(D-\bar{D})$ is equivalent to the maximum truck capacity which is still in the feasible solution area. Then, the variables from the deterministic demand model in (1) are used in (27) to represent the uncertain demand model. These variables give the solutions in the worst case under the constraint (28) determines as ellipsoidal uncertainty set.

The robust counterpart in Robust Optimization (RO) is provided by AIMMS software, which has been more recently applied to handle parameter uncertainty. RO is designed to meet some major challenges associated with uncertainty-affected optimization problems; to operate under lack of full information on the nature of uncertainty, to model the problem in a form that can be solved efficiently, and to provide guarantees about the performance of the solutions. RO is an uncertainty modeling approach suitable for a situation where the uncertainty ranges are known while the distribution not necessarily is. Typically, some inputs take an uncertain value anywhere between fixed minimum and maximum. This demand uncertainty can present how the worst case is when we consider the fluctuation of the demand. RO is very suitable for many problems as only simple inputs about data uncertainty are required as there are no scenarios or distribution functions need to be defined. The advantage of RO models is that they grow only slightly when uncertainty is added. As a result, the model can be solved efficiently.

The objective function containing the integer variables, is called Mixed Integer Program (MIP). In addition, one constraint is quadratic, thus the model becomes a Mixed Integer Program Quadratically-Constrained Program (MIQCP).

Whenever the uncertainty set of a mixed-integer robust problem is an ellipsoidal, the robust counterpart can be reformulated as a mixed-integer second-order cone program (MISOCP) which is a special case of MIQCP. This study uses a well-known powerful solver namely Gurobi (GUROBI website)[20], available for MISOCP. The algorithm used by GUROBI solvers is parallel SOCP barrier algorithm.

5. COMPUTATINAL RESULTS

Results illustrate outcomes for both deterministic and uncertain model. The anticipation results for both circumstances are the location of the facility and the total delivery cost of the three different network structures. As mentioned before, each network structure includes five demand scenarios, corresponding to five expectation results for each. We compare the total delivery cost of the three networks and indicate the best network structure by cost performance and network robustness. Then, we present the sensitivity analysis and compare the robustness of the three networks. Furthermore, we choose the best among the three networks to establish the vehicle routing designs.

The locations of opened facility for each network of both deterministic demand and uncertain demand are indicated in Table 1. From the results, we found that the network configurations and their systems have an impact on the total delivery cost on both deterministic demand and uncertain demand (see Figure 6). It can be seen that network 2 and network 3, which are defined for two layers of facility, are obviously preferable in term of cost performance when compared to network 1, which is single layer. The average total delivery costs of the three networks are, respectively 35,695,605 yen, 28,639,608 yen, and 29,552,784 yen in case of deterministic demand and 39,265,851 yen, 32,226,419 yen, and 33,054,993 yen in the uncertain demand case. Network 2 and network 3 lessened by 21.94 percent and 18.83 percent respectively in the deterministic case. Similarly, they also lessened by 19.69 percent and 17.18 percent in the uncertain case. The total delivery cost is mostly generated by the travel cost (approximately more than 90 percent), and its rapid increase is due to the amount of transportation.

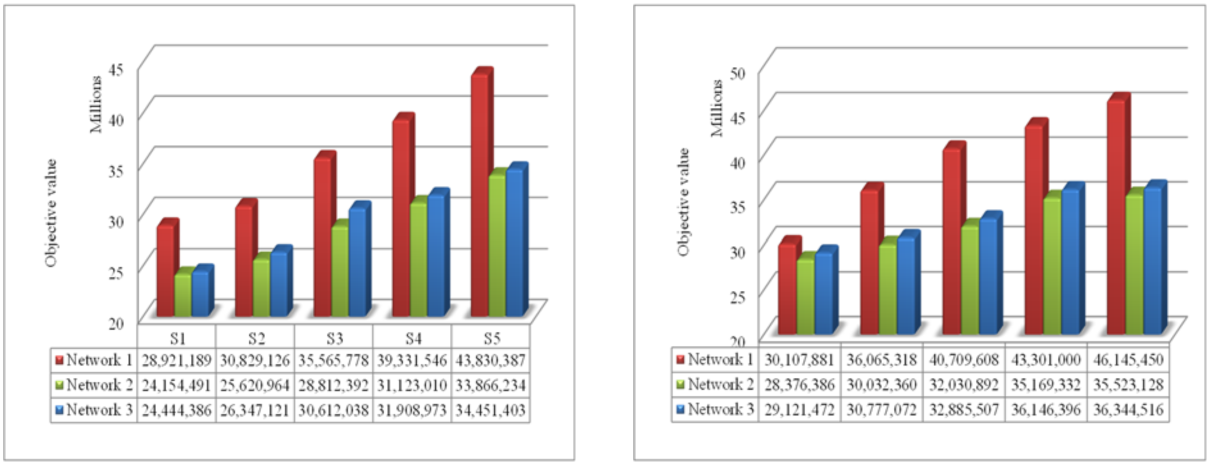
When comparing network 2 and network 3, all demand scenarios in network 2 can be reduced by 1.19 percent, 2.79 percent, 6.06 percent, 2.49 percent, and 1.71 percent, respectively. These results indicate that not only network configurations, but also truck size operations have significant influence on the total delivery cost function. Using 10-ton trucks to deliver from suppliers to central depots and from central depots to depots has a significant positive effect on cost reduction. Therefore, we can infer that network 2 meets with the criteria of cost performance.

Figure 7 illustrates the sensitivity analysis on the objective function for each network. In the range of sensitivity values comparison, network 1 presents a wider range and higher sensitivity than the others, meaning that network 1 is less robust than the other networks. The range scale of network 1 was approximately 0.1 to 9 million yen, while there is approximately 3 to 5 million yen of sensitivity for network 2 and network 3. In particular, when comparing the average sensitivity, network 1 is at 4,708,289 and 4,943,401 in term of deterministic and uncertain demand, while the corresponding values for network 2 are only 3,001,504 and 2,495,848. Moreover, the average sensitivity of network 2 is also lower than network 3, which is 3,325,625 and 2,552,370 for deterministic and uncertain demand, respectively. These results

Table 1 The opened facility locations for each network of both deterministic demand (left) and uncertainty demand (right)

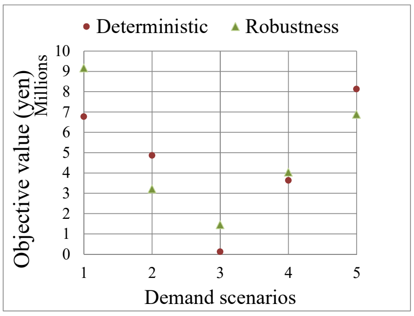
	Network														
	Network 1					Network 2					Network 3				
Central depot	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Kesennuma						1	1	1	1	1	1	1	1	1	1
Tome						1	1	1	1	1	1	1	1	1	1
Sendai(Kumagane)															
Yamamoto															
	Network 1					Network 2					Network 3				
Depot	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Tome and Kurihara			1	1	1			1	1	1			1	1	1
Kesennuma	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
North Ishinomaki	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
South Ishinomaki	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
South East Ishinomaki	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Taiwa Town															
North West Sendai															
North East Sendai															
Central Sendai															
South East Sendai															
South Sendai															

	Network														
	Network 1					Network 2					Network 3				
Central depot	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Kesennuma						1	1	1	1	1	1	1	1	1	1
Tome						1	1	1	1	1	1	1	1	1	1
Sendai(Kumagane)															
Yamamoto															
	Network 1					Network 2					Network 3				
Depot	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
Tome and Kurihara			1	1	1	1		1	1	1	1		1	1	1
Kesennuma	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
North Ishinomaki	1	1	1	1	1		1	1	1	1		1	1	1	1
South Ishinomaki	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
South East Ishinomaki	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Taiwa Town															
North West Sendai															
North East Sendai															
Central Sendai															
South East Sendai															
South Sendai															

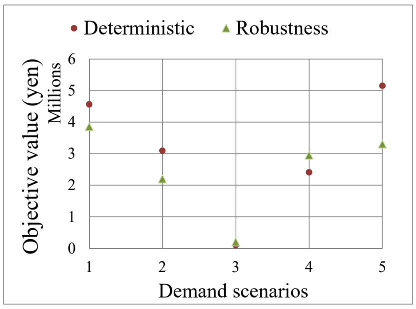


(a) Deterministic demand (b) Uncertainty demand

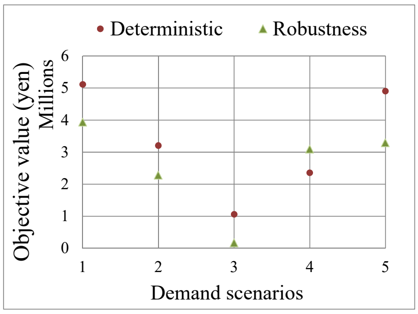
Fig.6 The total delivery cost



Network 1



Network 2



Network 3

Fig.7 The model sensitivity analysis of deterministic demand and uncertainty demand

indicate that network 2 is the most robust network among the three networks. In addition, the average sensitivity of uncertain demand for both network 2 and network 3 is more robust than in the deterministic case, showing the advantage of using RO.

In the comparison of deterministic demand and uncertain demand, network 2 and network 3 were similar that by using robust optimization to handle the uncertainty demand illustrated more robustness than ordinary deterministic demand. In addition, the fluctuation in sensitivity between deterministic demand and uncertain demand of network 2 was less than in network 3, meaning that network 2 is more robust than the other networks.

Moreover, we not only locate the optimal facility sites, but also allocate the optimal link flows. Results indicate that we can improve the best network among the three networks by making practical use of routing. According to the above comparisons, by considering cost efficiency and model robustness, network 2 offers the best outcome when the demand becomes uncertain. Consequently, we choose scenario 3 of this network as a representative to improve such an outcome through practical uses of vehicle routing problems. We focus on the last hierarchy of the network elements, from depots to shelters, and apply it to routing problems. The optimized routing network of each opened facility is illustrated in Figure 8. There are five depots selected at the optimal solutions, and each depot responds to its link flow allocation. The routing, which visits more than one shelter in one trip, consists instead of a single visit trip when link flows are less than truck capacity. We found that the total delivery cost is reduced for every opened facility site, by approximately 30 percent (Table 2).

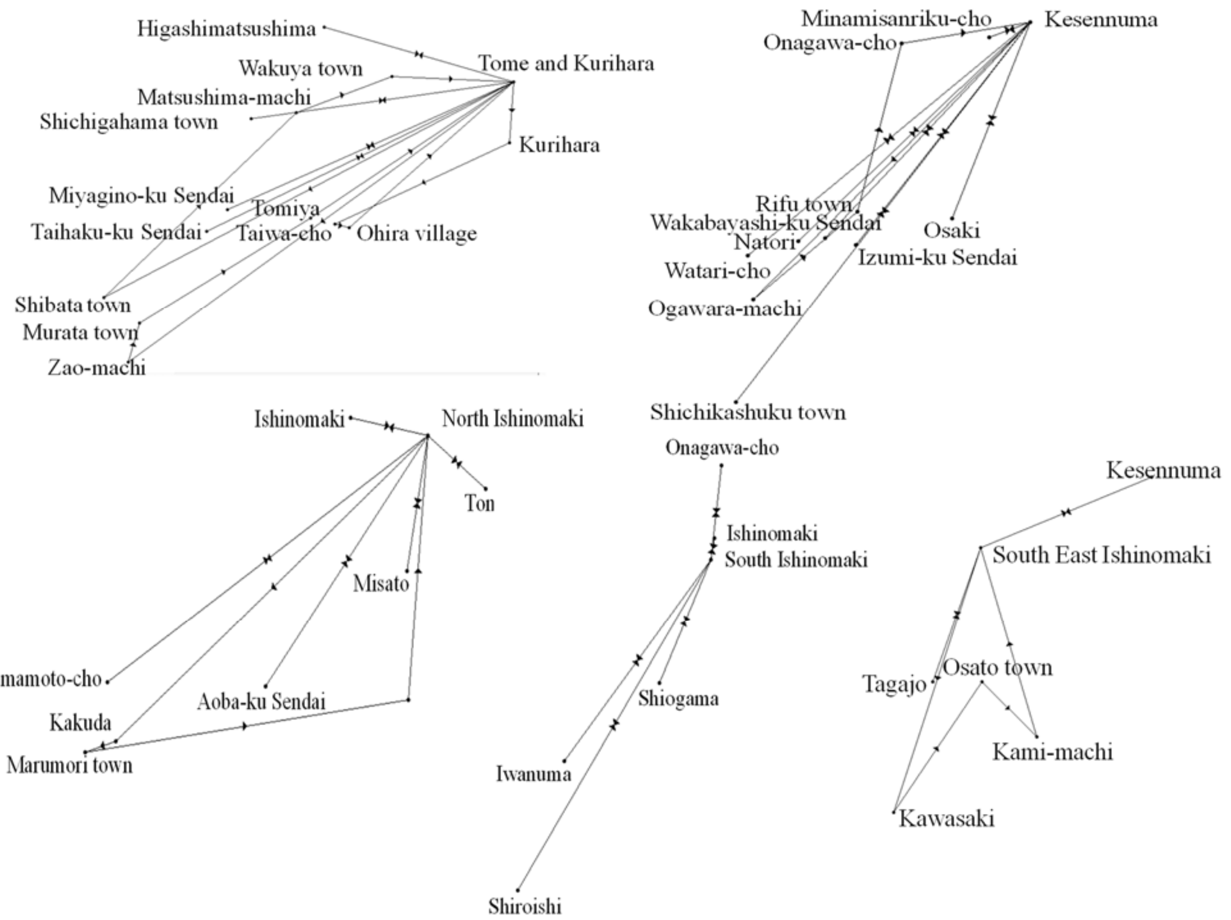


Fig.8 The routing operation of network 2, scenario

Table 2 The total delivery cost reduction by using LRP

Selected opening depots	Location problem of echelon 3	Improved by routing	Percentage of cost reduction
Tome and Kurihara	7,027,454	4,163,944	40.75
Kesennuma	6,640,056	3,704,803	44.21
North Ishinomaki	3,903,580	1,766,211	54.75
South Ishinomaki	2,731,894	1,739,353	36.33
South East Ishinomaki	3,124,383	2,587,476	17.18
Sum of echelon 3	23,427,368	13,961,787	
Travel cost of echelon 1 and 2	6,662,229	6,662,229	
Opening cost	677,732	677,732	
Transhipment cost	1,263,564	1,263,564	
Total delivery cost	32,030,892	22,565,312	29.55

6. CONCLUSION AND FUTURE WORKS

This study contributes to the previous research by considering the worst case scenarios of multi-facility location problems under uncertain demand. We justify the uncertainty on demand by the fact that it is quite difficult to predict post disaster demand. Therefore, our study determines the region of uncertain demand as an ellipsoid uncertainty set that is suitable for our situation where only the uncertainty ranges are known and not necessitate the distribution. Moreover, the ellipsoid uncertainty set is a novel approach that has never been fully applied to solve facility location problems. Accordingly, the main objective of the study is to tackle the facility locations and allocations with an uncertain demand function, as well as to improve the solution by integrating with vehicle routing problems. We analyze the three network structures including one single-layer facility network and two two-layer facility networks with distinct truck sizes (large trucks and small trucks). The uncertainty model helps the planner identify trade-offs between the inability to recover full costs for excess link flow and satisfying the demand.

First, the results from our calculation demonstrate that the network configurations influence total delivery costs. It can be seen clearly that the total delivery cost of network 2 and network 3 can reduce because the travel cost reduces, even though more facility cost and transshipment cost is required. Furthermore, results indicate that the travel cost has more significant influence than the facility opening cost. Moreover, truck size operation is important when the demand is high enough. This study found that large trucks are appropriate to deliver both inbound and outbound from the central depots. To apply the model, we suggest to establish the central depots and to using large trucks to deliver both inbound and outbound.

Furthermore, we show that networks are robust when the demand becomes uncertain or unknown. Here, we tested five different demand scenarios in each network based on the actual number of evacuees during post disaster. After solving for the uncertain demand by using robust optimization, the results prove that the structural networks have an effect on the model robustness. The two hierarchies for facilities provide more robustness than the single hierarchy case. Moreover, the uncertain demand model is more robust than the deterministic demand model. Accordingly, we conclude that the Robust Optimization has an advantage in solving uncertain models.

In addition, we improve the distribution network by routing design. The improved facility location with routing has an advantage in saving approximately 30 percent of the total delivery cost. Therefore, this study can help the decision maker plan for the efficient post disaster distribution network when demand turn out to be uncertainty.

The historical data was considered an example because of the simplicity entailed by the real geography. The model in this study represents the majority of demand. Therefore, the limitations of this study involve the data analysis and network assumptions. The results and findings are rather effects with this data. The two layers of facility network and the large truck size operation assumptions are probably an advantage when they match with the right demand. However, we tested the model with five demand scenarios, when they become less and high, and results from these assumptions and operations ensure that the model responds properly. The model provides the same trend of objective values. In addition, according to the model is MISOCP thus it is not able to guarantee global optimization. However, we use Gurobi solver, which has better performance[20].

Finally, we discuss the interrelated aspects to improve this work in the future as follows: (1) we are developing the integrated Location-Routing problem (LRP) explained by Hamidi et al.[21], in which the LRP integrates location, allocation, and also routing problems to design the network efficiency; (2) we have not considered other parameters that could fluctuate during humanitarian logistics, for example supply amount, unit transportation cost, facility opening cost, etc. Therefore, not only the uncertain demand but also such kind of parameters should be considered simultaneously; (3) future work could also consider the multi-objective facility location routing problem. The model should be more reasonable by investigating both cost and time indicators simultaneously. Once the uncertainty of demand is incorporated in the model, related robustness is evaluated.

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